

# Magnetoplasmon spectrum for asymmetric off-plane structure of dissipative 2D system

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The rigorous analysis of textbook result (Chiu and Quinn, 1974) gives unexpectedly a dramatic change of magnetoplasmon spectrum for asymmetric off-plane structure of dissipative 2D system. For given wave vector the dissipation enhancement leads to decrease(increase) of magnetoplasmon frequency at low(high) magnetic field. At certain dissipation strength the low-frequency plasmon excitation appears. In strong magnetic fields the magnetoplasmon frequency falls to cyclotron resonance line in presence of finite dissipation. The recent observation of 2D magnetoplasmon spectrum is consistent with our findings.

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## I. INTRODUCTION

Plasma oscillations in two-dimensional electron gas(2DEG) were first predicted in the middle 60th [1],[2],[3] and, then observed experimentally in liquid helium system [4] and silicon inversion layers [5],[6]. The observation [7] of magnetoplasmon(MP) spectrum reported to be affected by retardation effects, discussed more than tree decades ago[3], recommences the interest to the above problem. It was argued that in large-mesa 2D systems the role of edges becomes less significant, therefore the observed MP features can be accounted[8] within conventional theory[3] for unbounded 2D system. Unfortunately, some peculiar behavior of the magnetoplasmon spectrum cannot be explained within simple model of effective dielectric function of 2D system. In present paper we provide the rigorous analysis of magnetoplasmon spectrum taking into account the realistic off-plane asymmetry of 2D system. Our results are compared with experiments[7].

## II. 2D PLASMON DISPERSION LAW INFLUENCED BY DISSIPATION AND DIELECTRIC PERMITTIVITY MISMATCH

Let us assume 2D electron gas(see Fig.1), insert imbedded in a dielectric media with permittivities  $\epsilon_1$  and  $\epsilon_2$  of 1,2-halfspace respectively. In perpendicular magnetic field the complete set of Maxwell equations for in-plane components of the electrodynamic potentials  $\mathbf{A}$ ,  $\phi$  yield [9]

$$\begin{aligned} \square\phi &= 4\pi\rho, \square\mathbf{A} = \frac{4\pi\mathbf{j}}{c}, \\ \text{div}\mathbf{A} + \frac{\epsilon}{c}\frac{\partial\phi}{\partial t} &= 0, \\ \mathbf{j} &= -\hat{\sigma}\left(\nabla\phi + \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\right), \end{aligned} \quad (1)$$

where  $\square = \frac{\epsilon}{c^2}\frac{\partial^2}{\partial t^2} - \Delta$  is the d'Lambert operator. In presence of the magnetic field the components of conductivity

tensor  $\hat{\sigma}$  contains the longitudinal  $\sigma_{xx} = \sigma_{yy}$  and transverse  $\sigma_{yx} = -\sigma_{xy}$  components.

Assuming a certain magnetoplasmon  $e^{i\mathbf{q}\mathbf{r}-i\omega t}$  propagated in 2DEG, and, then separating longitudinal and transverse in-plane components of the vector potential[9], 2D magnetoplasmon dispersion relation yields:

$$\left[\frac{1}{4\pi}\left(\frac{\epsilon_1}{\kappa_1} + \frac{\epsilon_2}{\kappa_2}\right) + \frac{i\sigma_{xx}}{\omega}\right] \left[\frac{\kappa_1 + \kappa_2}{4\pi} - \frac{i\omega\sigma_{xx}}{c^2}\right] + \frac{\sigma_{yx}^2}{c^2} = 0, \quad (2)$$

where  $\kappa_{1,2} = \sqrt{q^2 - \epsilon_{1,2}\frac{\omega^2}{c^2}} > 0$  denotes the inverse penetration length of electromagnetic fields  $\sim e^{-\kappa|z|}$  into 1(2)-halfspace respectively. Note that Eq.(2) was first derived by Chiu and Quinn[3] and usually analyzed in shortcut form for off-plane symmetric 2D sample.

Let us specify the components of the conductivity tensor imbedded into Eq.(2). Following conventional Drude formalism one can represent them as it follows

$$\sigma_{xx} = \frac{i\tilde{\Omega}}{\tilde{\Omega}^2 - \Omega_c^2} \frac{\sigma_0}{\sigma}, \quad \sigma_{yx} = \frac{i\Omega_c}{\tilde{\Omega}} \sigma_{xx}, \quad (3)$$

where  $\sigma_0 = ne^2\tau/m$  is the zero-field Drude conductivity,  $n$  is the density,  $m$  is the effective mass, and  $\tau$  is the momentum relaxation time. Then, for actual problem of plasmon spectrum we use the notations made of use in Ref.[8]. Namely, we specify the dimensionless frequency  $\Omega = \frac{\omega}{\omega_p}$  and cyclotron frequency  $\Omega_c = \frac{\omega_c}{\omega_p}$  scaled by dimensional unit  $\omega_p = \frac{2\pi ne^2}{mc}$ . In addition, we use auxiliary quantity  $\tilde{\Omega} = \Omega + i/\sigma$ , where the dimensionless dissipation parameter  $\sigma = \frac{2\pi\sigma_0}{c}$  is known to be a measure of charge relaxation dynamics in presence of retardation effects[10, 11]

In absence of magnetic field, i.e. when  $\sigma_{yx} = 0$ , Eq.(2) decouples. The left(right)-hand term in square brackets defines the dispersion law for longitudinal(transverse) plasmon respectively. The primary interest concerns the dispersion law

$$\frac{\epsilon_1}{\kappa_1} + \frac{\epsilon_2}{\kappa_2} = \frac{2}{\Omega\tilde{\Omega}} \quad (4)$$

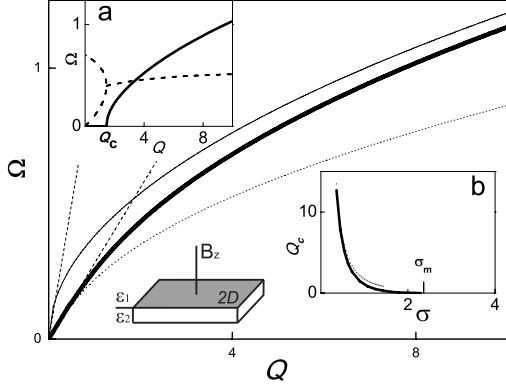


FIG. 1: Bold line represents the plasmon dispersion for dissipationless GaAs-based 2D structure ( $\epsilon_1 = 1, \epsilon_2 = 12.8$ ). Left(right)-hand dashed lines depict the light dispersion  $\Omega = Q/\sqrt{\epsilon_1}$  and  $\Omega = Q/\sqrt{\epsilon_2}$  respectively. Thin line depicts high- $Q$  asymptote  $\Omega = \sqrt{\frac{2Q}{\epsilon_1 + \epsilon_2}}$ . Dotted line corresponds to symmetric 2D structure  $\epsilon_1/\epsilon_2 = 1$ . Insert (a): solid(dashed) line depicts  $\Omega'$ ( $|\Omega''|$ ) part of the complex frequency for lossy 2D system at  $\sigma = 0.9$ . Insert (b): critical wave vector  $Q_c$  vs dissipation parameter  $\sigma < \sigma_m = 2.3$ . Dotted line depicts high- $Q$  asymptote. The art picture depicts the asymmetric 2D structure.

for longitudinal mode which could demonstrate weakly damped or purely relaxational behavior dependent on dissipation strength. Here,  $\kappa_{1,2} = \sqrt{Q^2 - \epsilon_{1,2}\Omega^2}$  is the dimensionless inverse penetration length,  $Q = \frac{qc}{\omega_p}$  the dimensionless wave vector.

We emphasize that the plasmon dispersion specified by Eq.(4) is affected by dielectric mismatch  $\epsilon_1 \neq \epsilon_2$ . To confirm this, we solve numerically Eq.(4) for typical GaAs-based 2D system[7] which exhibits strong dielectric mismatch since  $\epsilon_1 = 1, \epsilon_2 = 12.8$ . Let us first consider the dissipationless carriers case, when  $\sigma \rightarrow \infty$ . The result is represented by bold line in Fig.1, main panel. At high values of the wave vector  $Q \gg \sqrt{\epsilon_{1,2}}\Omega$  the retardation effects can be ignored. The plasmon spectrum(see thin line in Fig.1, main panel) obeys the familiar square-root dispersion relationship  $\Omega = \sqrt{\frac{2Q}{\epsilon_1 + \epsilon_2}}$  with average permittivity imbedded. This intuitive result is known in literature and claimed to be universal. We argue, however, that in the opposite low- $Q$  case the average permittivity scenario fails to account the plasmon spectrum. Indeed, at  $Q \geq \sqrt{\epsilon_{1,2}}\Omega$  the retardation effects becomes of primary importance. Consequently, the plasmon dispersion curve in Fig.1 is located well below the lowest light dispersion asymptote  $\Omega = Q/\sqrt{\epsilon_2}$  associated with GaAs bulk. We argue that the common use of "effective dielectric function of the medium" is well justified for high- $Q$  plasmon excitations[2] only. In opposite low- $Q$  case the retardation effects are of great importance, therefore one obliged to solve exactly the problem of electromagnetic fields in the vicinity of 2D system.

We now examine the plasmon spectrum for real case

of dissipative 2D system. At fixed value of dissipation strength the solution of Eq.(4) gives of the complex frequency  $\Omega = \Omega' + i\Omega''$ . The plasmon is damped when  $\Omega'' < 0$ . We verify that for arbitrary value of the dissipation the plasmon electromagnetic fields are indeed localized within 2D plane since  $\text{Re}(\kappa_{1,2}) > 0$ .

As an example, in Fig.1,a we plot both the real and imaginary part of the frequency vs plasmon wave vector for certain value of dissipation strength  $\sigma = 0.9$ . The evidence shows that the complex frequency becomes positive when  $Q > Q_c$ , where  $Q_c$  is a certain critical wave vector. In general, the critical wave vector can be represented as a function of  $\sigma$ , and  $\epsilon_{1,2}$ . For actual 2D system in question we find numerically the critical diagram  $Q_c(\sigma)$  represented by the bold line in Fig.1,b. Fortunately, the above dependence can be found analytically within high- $Q$  non-retarded limit. Indeed, with the help of Eq.(4) the real and imaginary parts of frequency yield

$$\Omega' = \sqrt{\frac{2Q}{\epsilon_1 + \epsilon_2} - \frac{1}{4\sigma^2}}, \quad \Omega'' = -\frac{1}{2\sigma}. \quad (5)$$

Eq.(5) provides the critical diagram asymptote  $Q_c(\sigma) = \frac{\epsilon_1 + \epsilon_2}{8\sigma^2}$  shown by dotted line in Fig.1,b. Note that the critical diagram  $Q_c(\sigma)$  demonstrates vanishing at certain magnitude of the dissipation  $\sigma_m$ . Remarkably, the analytic approach allows us to find the above value as well. Indeed, the substitution  $Q, \Omega' = 0, \Omega'' \rightarrow 0$  into Eq.(4) gives the result  $\sigma_m = \frac{\sqrt{\epsilon_1 + \epsilon_2}}{2}$ . For symmetric case  $\epsilon_{1,2} = \epsilon$  our finding coincides with that  $\sigma_m = \sqrt{\epsilon}$  reported in Ref.[9].

### III. DRAMATIC CHANGE OF MAGNETOPLASMON SPECTRUM CAUSED BY OFF-PLANE DIELECTRIC ASYMMETRY AND DISSIPATION

The up-to-date attempts to analyze the plasmon spectrum in presence of magnetic field concern either non-dissipative [3] or dissipative 2D plasma[12]. In both cases the dielectric mismatch of 2D structure was neglected. We now demonstrate that both the off-plane dielectric asymmetry and dissipation strongly affect the magnetoplasmon spectrum.

Substituting the components of conductivity tensor specified by Eq.(3) into Eq.(2) the magnetoplasmon spectrum yields

$$\Omega_c = \tilde{\Omega} \left[ \frac{\left( \frac{\epsilon_1}{\kappa_1} + \frac{\epsilon_2}{\kappa_2} - \frac{2}{\Omega\tilde{\Omega}} \right) \left( \kappa_1 + \kappa_2 + \frac{2\tilde{\Omega}}{\Omega} \right)}{\left( \frac{\epsilon_1}{\kappa_1} + \frac{\epsilon_2}{\kappa_2} \right) (\kappa_1 + \kappa_2)} \right]^{1/2}. \quad (6)$$

Evidently, the zero-field dispersion law specified by Eq.(4) follows from Eq.(6) when  $\Omega_c = 0$ .

We argue that for fixed values of dielectric permittivities  $\epsilon_{1,2}$ , wave vector  $Q$  and dissipation strength  $\sigma$  one can solve Eq.(6) and, then find the real  $\Omega'$  and imaginary  $\Omega''$  components of the frequency as a function of

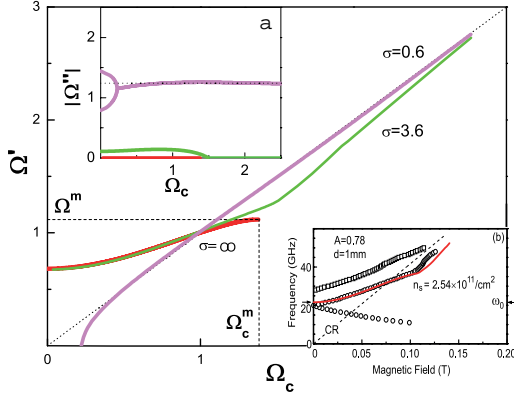


FIG. 2: Main panel(insert a) shows the real(imaginary) parts respectively of the complex magnetoplasmon frequency vs dimensionless cyclotron frequency at fixed wave vector  $Q = 4$  for dissipationless  $\sigma = \infty$  and lossy  $\sigma = 3.6; 0.6$  GaAs-based 2D system( $\epsilon_1 = 1, \epsilon_2 = 12.8$ ). The magnetoplasmon cutoff for clean 2D gas occurs at  $\Omega^m = 1.12$  and  $\Omega_c^m = 1.38$ . Dotted lines depicts high-field asymptotes given by Eq.(8) for 2D system of highest dissipation strength. Insert b: Experimental data[7] for disk-shaped GaAs sample. Red line depicts the result of calculations for  $Q = 4$  and  $\sigma = 5$ .

cyclotron frequency  $\Omega_c$ . It is useful to find the magnetoplasmon spectrum under the progressive growth of dissipation. Let us examine first the dissipationless carriers case when magnetoplasmon is undamped. Substitution  $\Omega'' = 0$  into Eq.(6) readily gives the sought-for spectrum as  $\Omega_c(\Omega', Q)$ . The result is plotted in Fig.2. Remarkably, the lossless magnetoplasmon excitation exhibits a certain cutoff point  $\Omega^m, \Omega_c^m$  on the spectrum plot. Indeed, for asymmetric 2D structure in question the magnetoplasmon penetration length scale becomes infinitely large for 2-halfspace when  $\kappa_2 = 0$ . Under this condition Eq.(6) provides the cutoff point as  $\Omega^m = \frac{Q}{\sqrt{\epsilon_2}}$

and  $\Omega_c^m = \left[ \frac{Q^2}{\epsilon_2} + \frac{2Q}{\sqrt{\epsilon_2 - \epsilon_1}} \right]^{1/2}$  respectively. Note, for symmetric 2D structure the magnetoplasmon spectrum demonstrates the saturation at high magnetic fields[3, 8] therefore  $\Omega_c^m \rightarrow \infty$ . We now examine the magnetoplasmon spectrum assuming finite dissipation of 2D system. In general, Eq.(6) can be solved numerically. In Fig.2 we represent the result of numerical calculations for fixed wave vector and different values of the dissipation strength. It can be seen that increase of dissipation results in dramatic change of magnetoplasmon spectrum. Firstly, at low magnetic fields the real(imaginary) component of the complex frequency decreases(increases) under enhancement of dissipation. At  $B = 0$  and fixed wave vector  $Q$  the graphic solution  $Q_c(\sigma_c) = Q$  of critical diagram shown in Fig.1b denotes the critical value of dissipation  $\sigma_c < \sigma_m$ . The small departure from critical value  $\sigma \leq \sigma_c$  results in abrupt change of the plasmon frequency from a finite value to zero. The latter can be assigned as low-frequency mode. Further degradation of 2D sys-

tem  $\sigma \rightarrow 0$  leads to shift of low-frequency mode towards higher magnetic fields. Low-frequency mode threshold appears. The threshold behavior of low-frequency mode is clearly seen in Fig.2.

In the opposite case of high magnetic fields the behavior of the magnetoplasmon spectrum is striking as well. At first, the set of curves plotted for different dissipation strengths exhibits a certain crossing point at cyclotron resonance line  $\Omega_c = \Omega'$ . Substituting this condition into Eq.(6) one obtains the transcendental equation

$$\frac{\epsilon_1}{\kappa_1} + \frac{\epsilon_2}{\kappa_2} = \frac{\kappa_1 + \kappa_2 + 2}{\Omega^2}. \quad (7)$$

which allows to find the sought-for frequency associated with the crossing point. Well above the crossing point the magnetoplasmon spectrum demonstrates the change from cutoff trend owned to dissipationless case to linear in magnetic field behavior for lossy magnetoplasmon. It is to be noted that for moderate dissipation  $\sigma \sim 1$  the magnetoplasmon spectrum exhibits a non-monotonic behavior seen in Fig.2. To confirm the predictions of our model, in Fig.2,b we analyze the experimental data[7]. For actual 2D density  $n = 2.54 \times 10^{11} \text{cm}^{-2}$  and mesa diameter  $d = 1 \text{mm}$  of GaAs sample, we find the frequency  $\omega_p = 2 \times 10^{11} \text{c}^{-1}$  and wave vector  $q = 2.4/d = 24 \text{cm}^{-1}$ [7]. The best fit of the data shown in Fig.2,b by the red line corresponds to dimensionless wave vector  $Q = 4$  and dissipation  $\sigma = 2$ . The later allows one to estimate the carrier mobility  $\sim 0.3 \times 10^6 \text{cm}^2/\text{Vs}$  being close to that  $\sim 10^6 \text{cm}^2$  reported in experiment.

Remarkably, for finite dissipation strength we are able to solve analytically Eq.(6) within high frequency limit  $\Omega \gg Q/\sqrt{\epsilon_{1,2}}$ . Indeed, using the expansion for inverse penetration length  $\kappa_{1,2} = i\sqrt{\epsilon_{1,2}}\Omega \left( 1 - \frac{Q^2}{2\Omega^2\epsilon_{1,2}} \right)$  we find both the real and imaginary components of the complex frequency as it follows

$$\Omega' = \Omega_c, \quad \Omega'' = -\frac{1}{\sigma} + \frac{1}{\sigma_m}. \quad (8)$$

Note that Eq.(8) is valid when  $\sigma < \sigma_m$ . In Fig.2 we plot the asymptotes specified by Eq.(8) for fixed  $\sigma = 0.6$ . Evidently, at high magnetic fields the numerical data is well described by the above asymptotes. It is quite interesting that plasmon wave vector is dropped out in Eq.(8). Therefore, for arbitrary wave vectors and certain dissipation strength one expects the same complex frequency of magnetoplasmon at high magnetic fields. We underline also that at high magnetic field the magnetoplasmon is localized nearby 2D plane. Indeed, Eq.(8) allows one to find the correct inverse penetration length of electromagnetic fields on both sides of 2D plane, i.e.  $\text{Re}(\kappa_{1,2}) \simeq \sqrt{\epsilon_{1,2}}|\Omega''| > 0$ .

#### IV. CONCLUSIONS

Our analysis of textbook Eq.(2) derived in early 70-s[3] provides a strong doubts concerning overall use of effec-

tive dielectric function approach. In contrast, we demonstrate the dramatic change of magnetoplasmon spectrum for realistic case of asymmetric off-plane structure of dissipative 2D systems. Under the growth of dissipation a magnetoplasmon with a certain wave vector demonstrates the decrease (increase) of its frequency at low (high) magnetic field. At certain dissipation strength the low-frequency plasmon mode appears first at  $B = 0$ .

At high magnetic fields the real(imaginary) component of magnetoplasmon frequency follows the cyclotron resonance asymptote and depends on the dissipation strength respectively. For moderate dissipation strength our calculations provide an evidence of non-monotonic behavior of magnetoplasmon spectrum already observed in experiment.

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